

The Epistemic Logic of Friendship

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Joint work with

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Outline

- 1 Introduction
- 2 Epistemic logic: a quick summary
- 3 Epistemic logic of friendship
- 4 PDL-transformations: a quick summary
- 5 Social announcements
- 6 GDDL and privacy
- 7 Knowing your friends
- 8 Common knowledge reconsidered
- 9 Conclusion

Reasoning about

- knowledge of social relationships
- knowledge on the basis of social relationships
- communication via social relationships
- (changes to social relationships)

Reasoning about

- knowledge of social relationships
Which friends of mine does she know about?
- knowledge on the basis of social relationships
- communication via social relationships
- (changes to social relationships)

Reasoning about

- knowledge of social relationships
Which friends of mine does she know about?
- knowledge on the basis of social relationships
What do her friends know?
- communication via social relationships
- (changes to social relationships)

Reasoning about

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Which friends of mine does she know about?
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If she told her friends, would I know?
- (changes to social relationships)

Reasoning about

- knowledge of social relationships
Which friends of mine does she know about?
- knowledge on the basis of social relationships
What do her friends know?
- communication via social relationships
If she told her friends, would I know?
- (changes to social relationships)
If he became her friend, what would he know?

Spies in danger

Berlin, 1978. A spy network has recently been uncovered by the Stasi, who are rounding up the spies and their associates. Bella (*b*) is friends with Charlie (*c*) and Erik (*e*), neither of whom are friends with each other. Unknown to the others is that Erik is a spy (*s*). The others are not spies, and Erik knows that because all spies know who else is a spy (we suppose). Bella knows that Charlie is not a spy, but Charlie does not know about her. After the network is exposed, all the spies and their friends will be interrogated by the police. But just before this happens a message is relayed to all agents revealing whether or not they are in danger, that is, whether they are a spy (which they would know in any case) or a friend of a spy. Who now knows that Erik is a spy?

Roger's Quandary

Peggy (p) knows that Roger (r) is cheating (c) on his wife, Mona (m). What's more, Roger knows that Peggy knows, because they met accidentally while he was with his mistress. Mona does not know about the affair, and both Peggy and Roger know this. The situation (for Roger) deteriorates when he discovers that Peggy is a terrible gossip. She is bound to have told all her friends about his affair. What Roger does not know is whether Mona is a friend of Peggy (she is). Who knows what, exactly?

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Epistemic logic: a quick summary

- Epistemic logic (S5): reasoning about knowledge

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Were it to be announced that Bob is in danger, Alice and Bob would know that.

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- Dynamic epistemic logic (DEL)

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It is common knowledge that Bob is in danger.
- Public announcement logic (PAL)
Were it to be announced that Bob is in danger, Alice and Bob would know that.
- Dynamic epistemic logic (DEL)
If Bob does not know that he is in danger, and Alice were told this in private, Bob would still not know it.

Epistemic logic: the symbols

$K_e(p \rightarrow q)$ Erik (e) knows that if (p) the network has been exposed then (q) there is a mole.

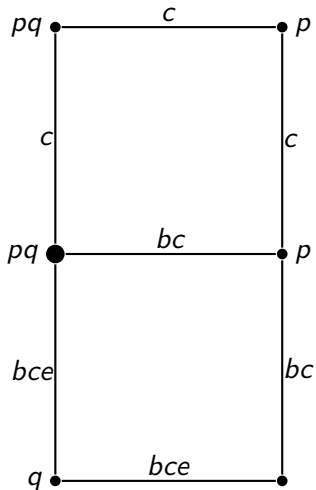
$K_b \neg K_c p$ Bella knows that Charlie (c) does not know the network has been exposed.

$C_{bc} p$ It is common knowledge among Bella and Charlie that the network has been exposed.

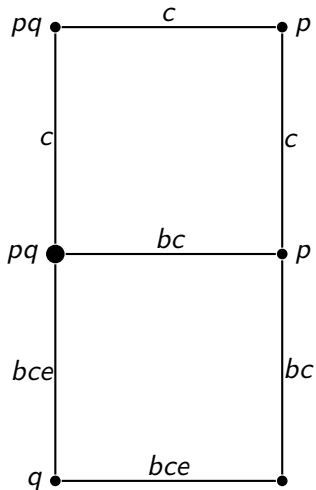
$[p!]K_e q$ If it were publicly announced that the network has been exposed, Erik would know there is a mole.

$(\neg K_e q \wedge \boxed{[p!] \xrightarrow{e} I} \neg K_e q)$ Erik doesn't know that there is a mole, and were it to be announced privately to the others that the network has been exposed, he would still not know.

Epistemic logic: the models



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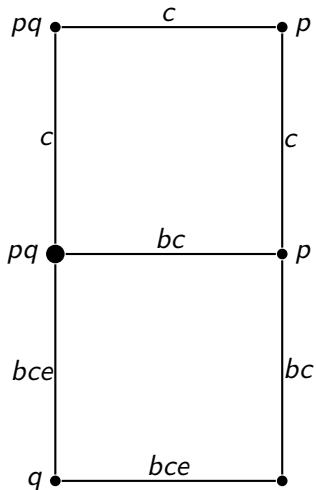
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$$C_{\{bc\}} p$$

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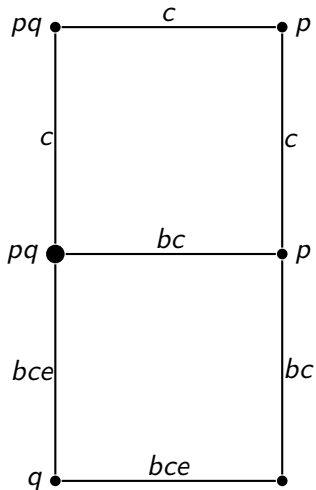
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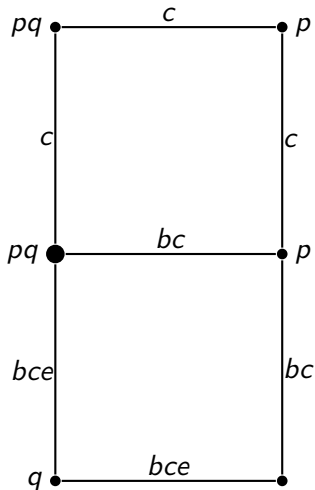
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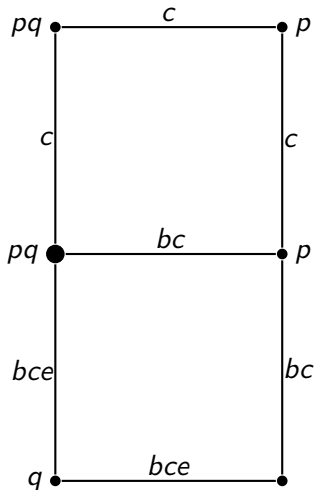
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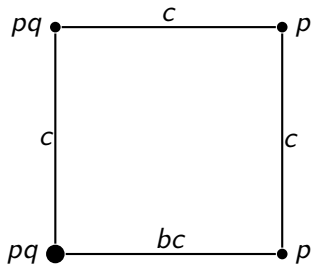
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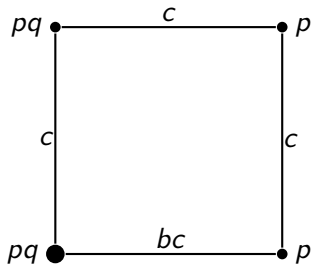
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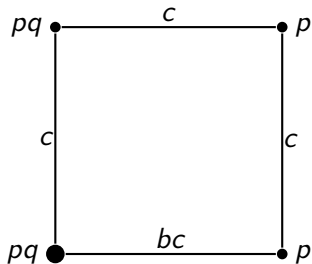
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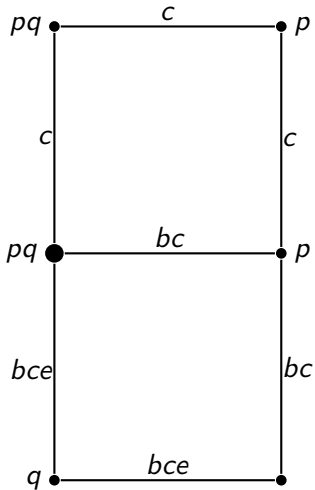
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$$[p!]C_{\{bce\}} p \quad \checkmark$$

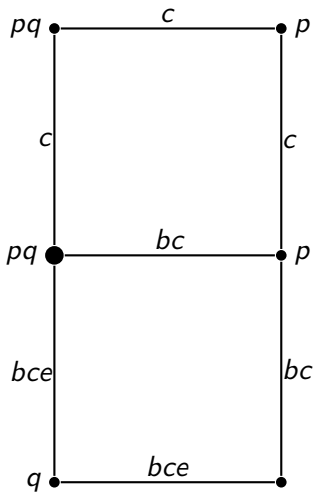
Epistemic logic: the models

$$(\neg K_e q \wedge \boxed{[p]} \xrightarrow{e} \boxed{!} \neg K_e q)$$



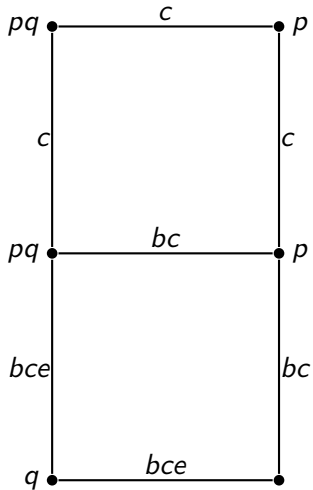
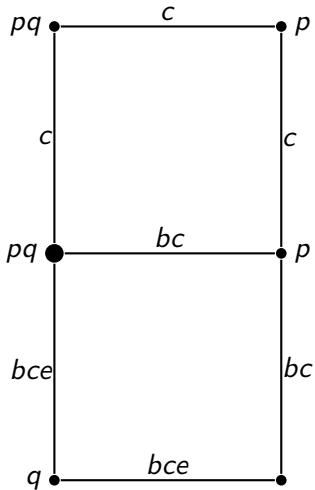
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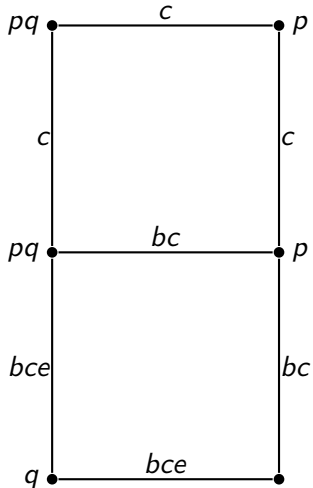
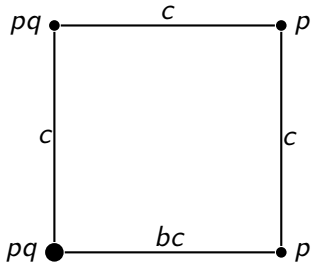
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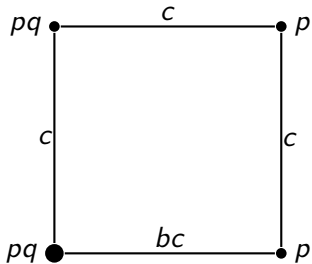
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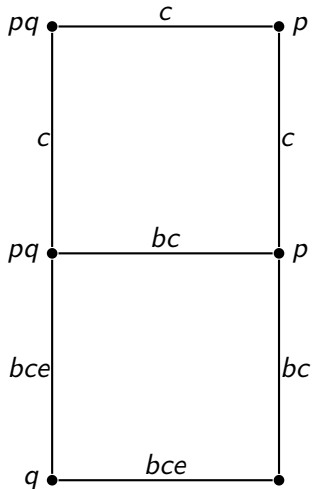


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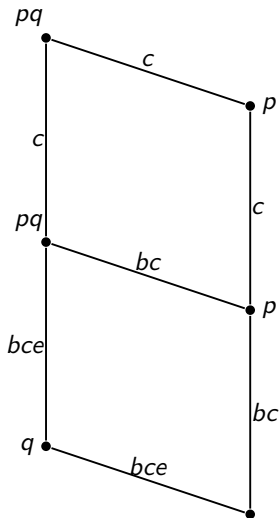
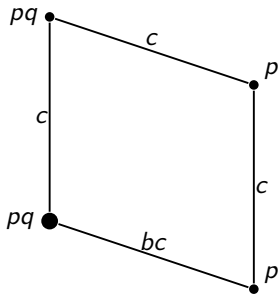


e



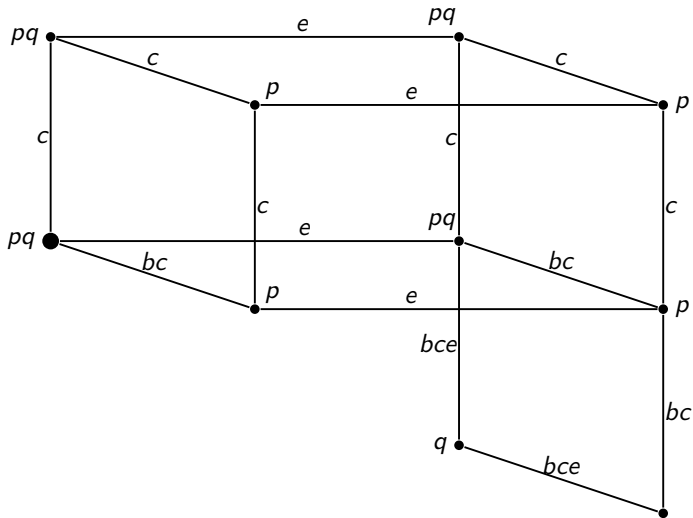
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$$(\neg K_e q \wedge \boxed{[p]} \text{---}^e \text{---} [] \neg K_e q)$$



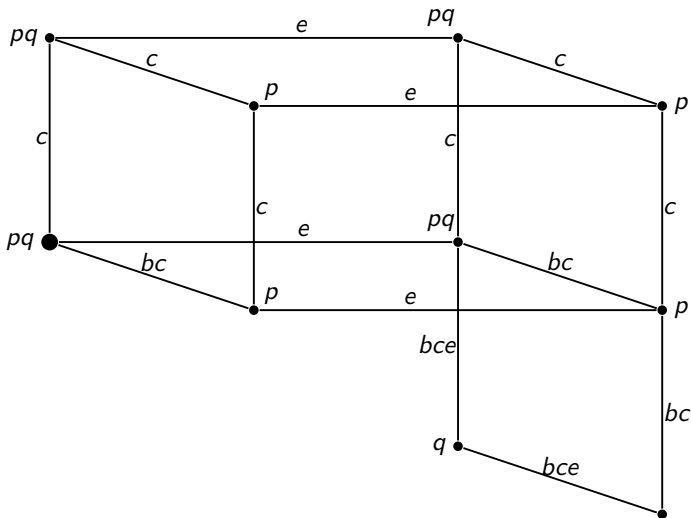
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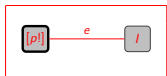


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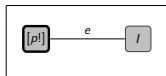
$$(\neg K_e q \wedge \boxed{[p]} \text{---}^e \text{---} I \neg K_e q) \checkmark$$



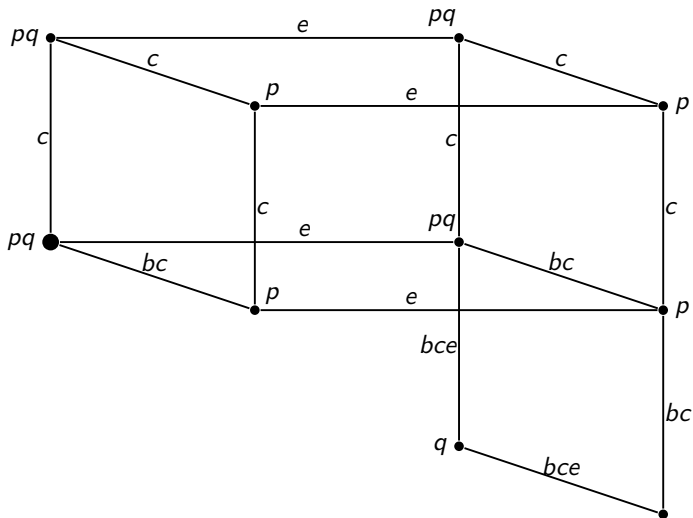
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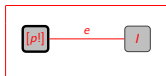
C_{bcP}



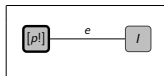
C_{beP}



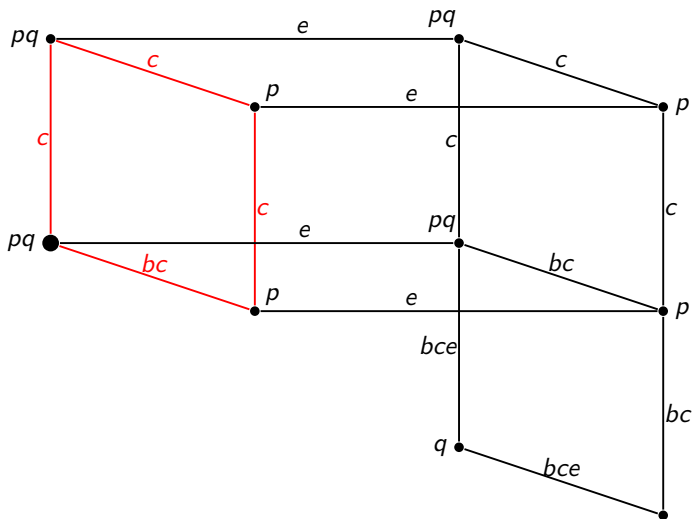
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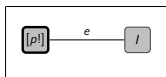
C_{bcP}



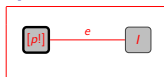
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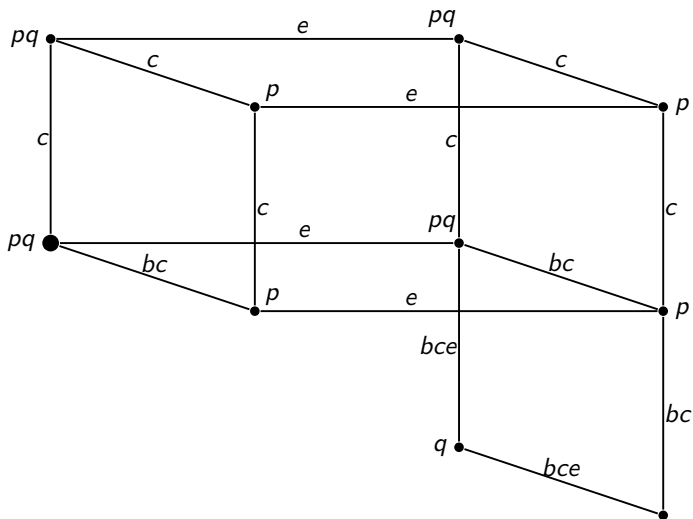
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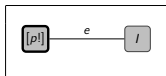
$C_{bc}P$ ✓



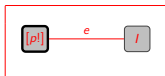
$C_{be}P$



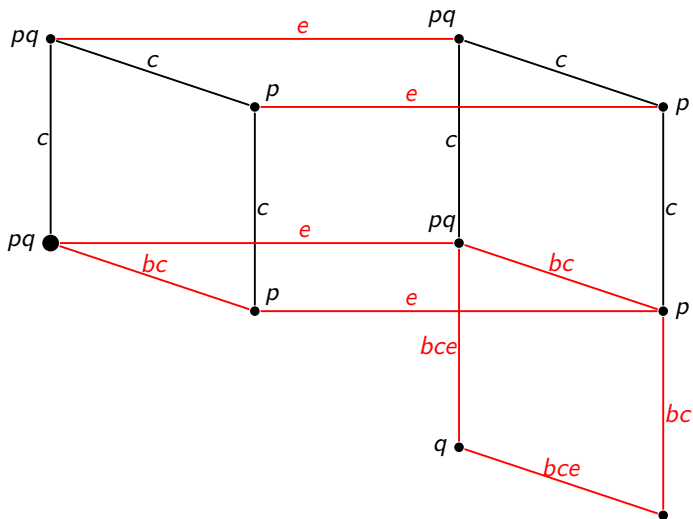
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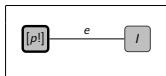
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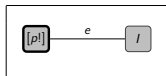
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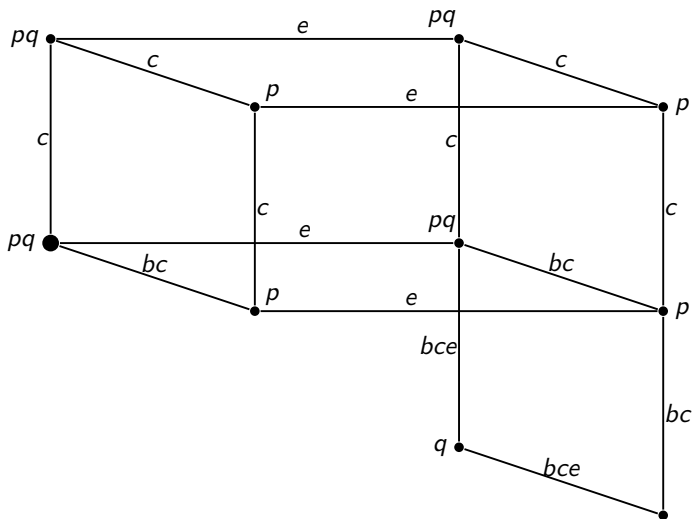
Epistemic logic: the models



$C_{bc}P \quad \checkmark$



$C_{be}P \quad \times$



Epistemic logic: syntax and semantics

Formulas

$\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid K_\alpha\varphi \quad (p \in \text{Prop}, \alpha \in A)$

Models

$M = \langle W, k, V \rangle$

k_a is an equivalence relation on W for each $a \in A$

$V(p) \subseteq W$ for each $p \in \text{Prop}$

Epistemic logic: syntax and semantics

Semantics

$M, w \models p$	iff	$w \in V(p)$
$M, w \models \neg\varphi$	iff	$M, w \not\models \varphi$
$M, w \models (\varphi \wedge \psi)$	iff	$M, w \models \varphi$ and $M, w \models \psi$
$M, w \models K_a\varphi$	iff	$M, v \models \varphi$ for all $v \in W$ such that $k_a(w, v)$
$M, w \models \Delta\varphi$	iff	$\Delta M, \Delta w \models \varphi$

where Δ is one of a range of dynamic operators

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Epistemic logic of friendship

Extend epistemic logic to EFL, an *epistemic logic of friendship*

Goal

To represent the logical structure of propositions and reasoning about knowledge in a social context, specifically those involving friendship, construed as a binary relation between agents.

Epistemic logic of friendship: quick summary

Indexical propositions

Naming agents

Friendship modality

Anaphora

$\neg d$	I am not in danger (d)
$\neg K @_e s$	I don't know that Erik (e) is a spy (s)
$F d$	All my friends are in danger
$K F d$	I know that all my friends are in danger
$@_e K F d$	Erik knows that all his friends are in danger
$F K d$	All my friends know they are in danger
$\langle F \rangle d$	Some of my friends are in danger
$\langle F \rangle c$	Charlie (c) is my friend
$\langle F \rangle K @_e d$	Some of my friends know that Erik is in danger
$\downarrow n \langle F \rangle K @_n d$	I have a friend who knows that I am in danger

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Epistemic logic of friendship: models

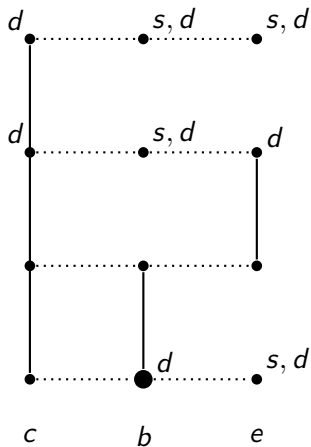
knowledge: ● — k — ●

friendship: ● f ●

Epistemic logic of friendship: models

knowledge: $\bullet \xrightarrow{k} \bullet$

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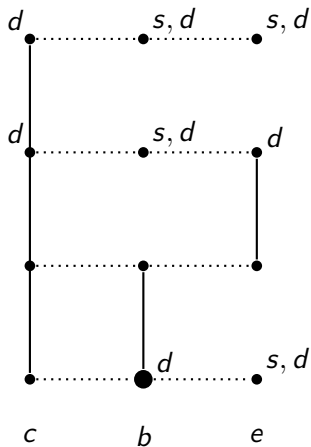


- $\neg d$
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- Fd
- $@_e K Fd$
- FKd
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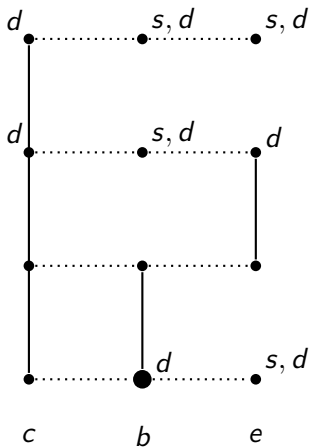


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$\neg d$ \times

$\neg K @_e s$

Fd

$@_e K Fd$

$F K d$

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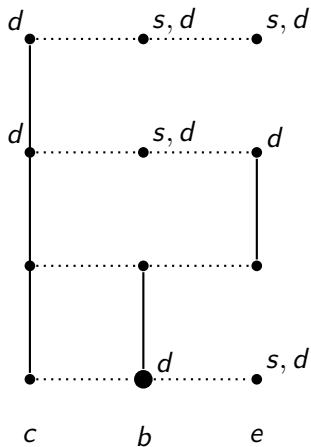
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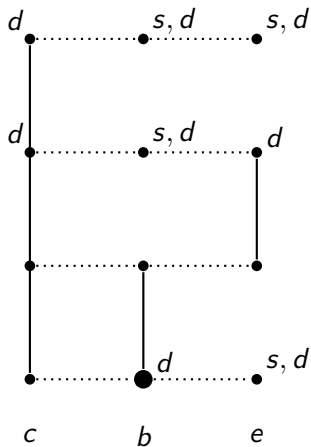
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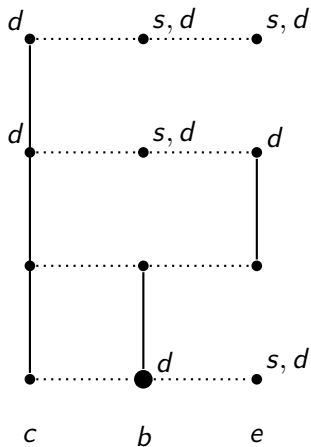


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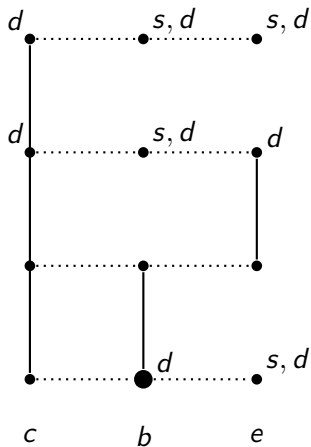
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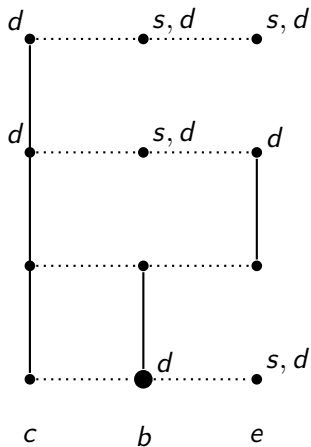


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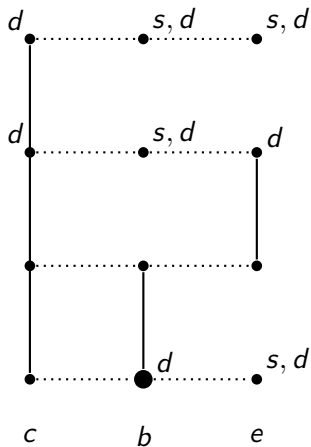


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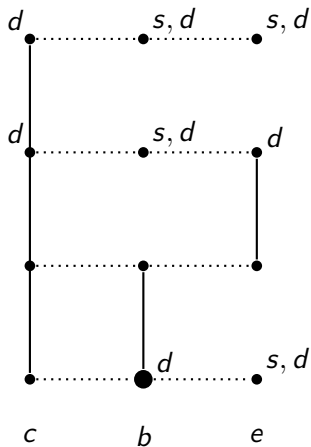


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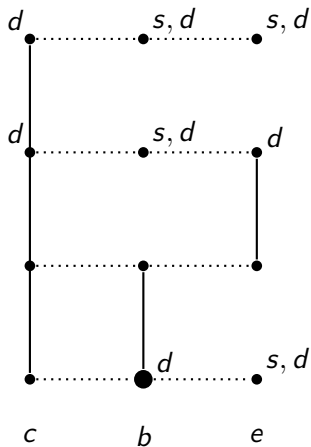


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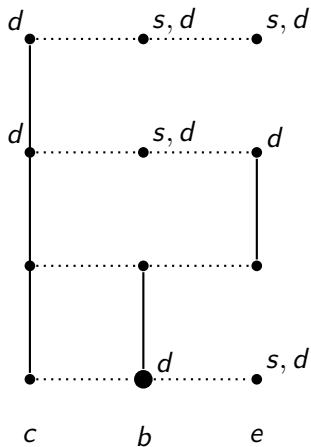


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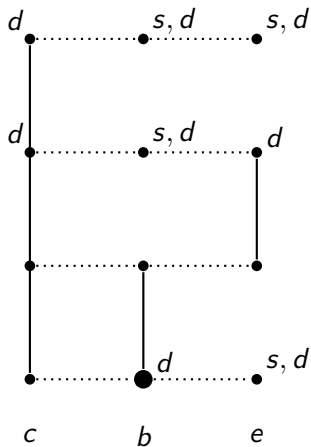
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I have a friend who does not know whether I am a spy

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$@_e KFd$	✓
FKd	×
$\langle F \rangle d$	✓
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$\downarrow_n \langle F \rangle K@_n d$	✓

I have a friend who does not know whether I am a spy

$\downarrow_n \langle F \rangle \neg (K@_n s \vee K@_n \neg s)$

Epistemic logic of friendship: syntax and semantics

Formulas

$\varphi ::= \rho \mid \eta \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K\varphi \mid F\varphi \mid A\varphi \mid \downarrow\eta \varphi$
($\rho \in \text{Prop}$, $\eta \in \text{ANom}$)

Models

$M = \langle W, A, k, f, g, V \rangle$

k_a is an equivalence relation on W for each $a \in A$

f_w is an irreflexive, symmetric relation on A for each $w \in W$

$g(\eta) \in A$ for each $\eta \in \text{ANom}$

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where Δ is one of a range of dynamic operators

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The @ operator and how to get rid of it

Semantics

$$M, w, a \models @_{\eta}\varphi \quad \text{iff} \quad M, w, g(\eta) \models \varphi$$

This can be defined, thanks to the following validity

$$@_{\eta}\varphi \quad \leftrightarrow \quad A(\eta \rightarrow \varphi)$$

The binder and how to get rid of it

Semantics

$$M, w, a \models \downarrow \eta \varphi \quad \text{iff} \quad M[\frac{\eta}{a}], w, a \models \varphi$$

M is a *named agent model*, if every agent in M has a name, i.e., for each $a \in A$, there is an $n \in \text{ANom}$ (finite) such that $g(n) = a$.

Valid in named agent models:

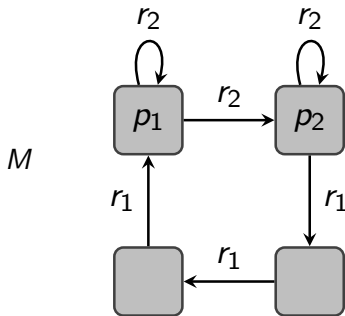
$$\downarrow n \varphi \quad \leftrightarrow \quad \bigvee_{m \in \text{ANom}} (m \wedge \varphi[\frac{n}{m}])$$

Outline

- 1 Introduction
- 2 Epistemic logic: a quick summary
- 3 Epistemic logic of friendship
- 4 PDL-transformations: a quick summary**
- 5 Social announcements
- 6 GDDL and privacy
- 7 Knowing your friends
- 8 Common knowledge reconsidered
- 9 Conclusion

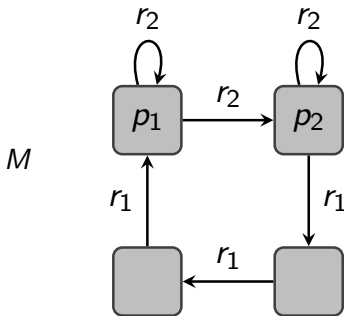
Defining relations with PDL

Within this model, we can define new relations using PDL.



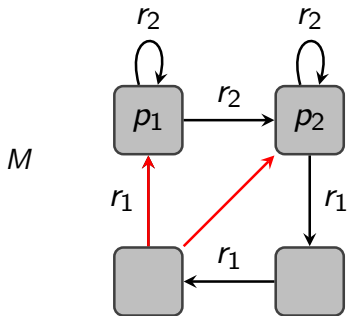
Defining relations with PDL: Composition

Composition: $r_1; r_2$



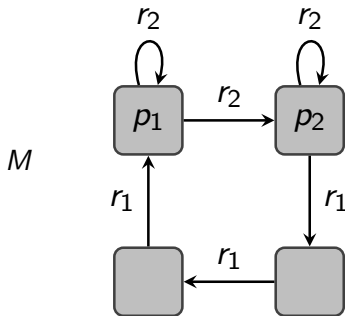
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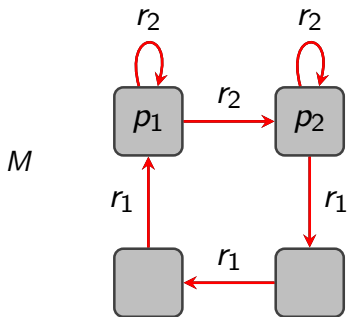
Defining relations with PDL: Choice

Choice: $r_1 \cup r_2$



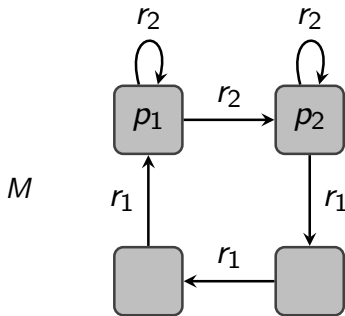
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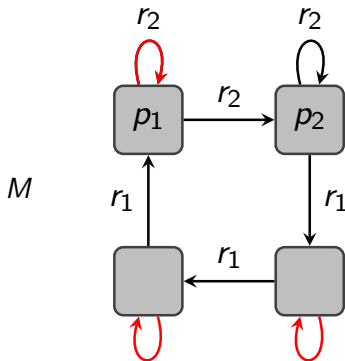
Defining relations with PDL: Test

Test: $\neg p_2?$



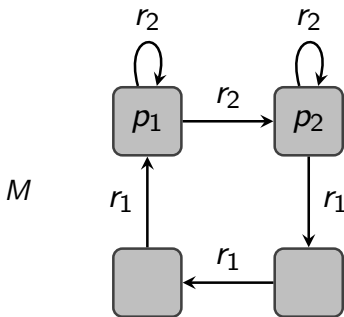
Defining relations with PDL: Test

Test: $\neg p_2?$



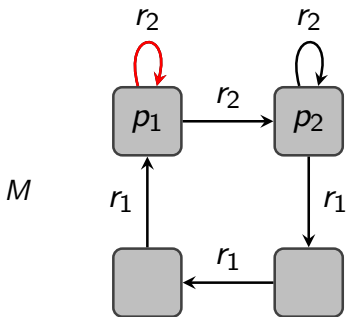
Defining relations with PDL: Iteration

Iteration: r_1^*



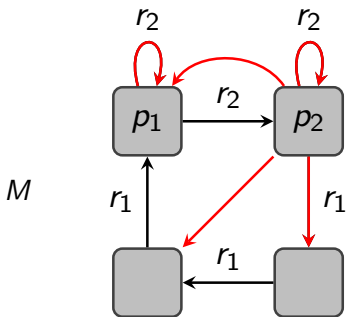
Defining relations with PDL: Iteration

Iteration: r_1^*



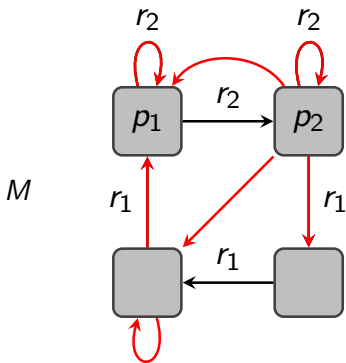
Defining relations with PDL: Iteration

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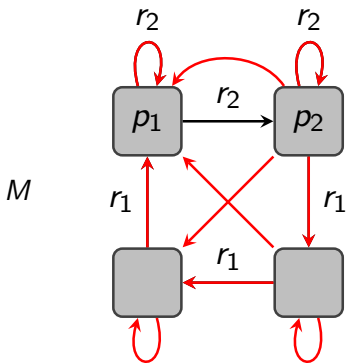
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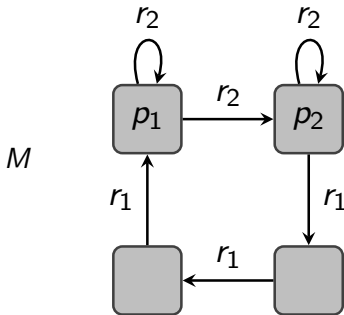
Defining relations with PDL: Iteration

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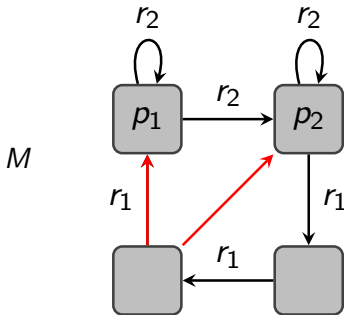
Defining relations with PDL: Programs

Program: $(r_1; r_2) \cup (p_1?; r_2)$



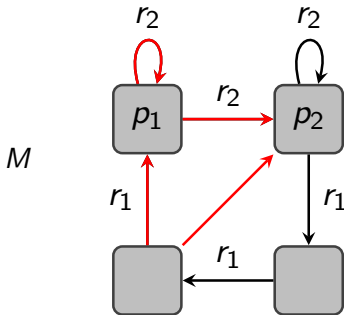
Defining relations with PDL: Programs

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Defining relations with PDL: Programs

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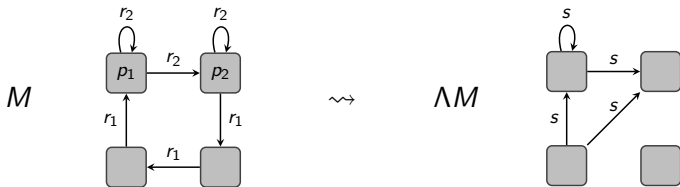


PDL-transformations

$$\Lambda(s) = (r_1; r_2) \cup (p_1?; r_2)$$

PDL-transformations

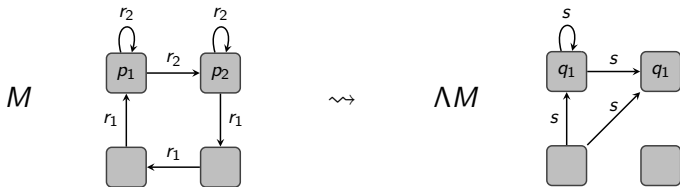
$$\Lambda(s) = (r_1; r_2) \cup (p_1?; r_2)$$



PDL-transformations

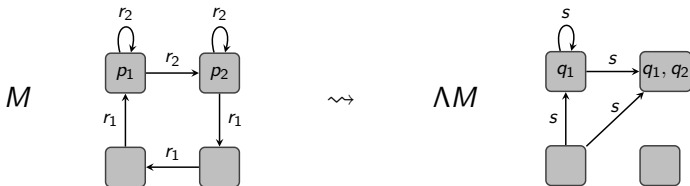
$$\Lambda(q_1) = \langle r_2 \rangle \neg p_1$$

$$\Lambda(s) = (r_1; r_2) \cup (p_1?; r_2)$$



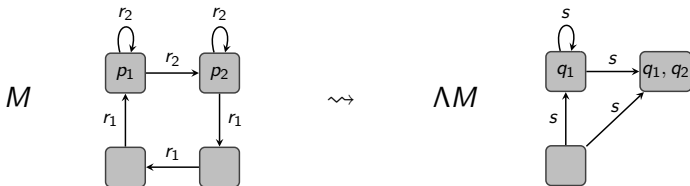
PDL-transformations

$$\begin{aligned}\Lambda(q_1) &= \langle r_2 \rangle \neg p_1 \\ \Lambda(q_2) &= \langle p_2?; r_1 \rangle \neg p_2 \\ \Lambda(s) &= (r_1; r_2) \cup (p_1?; r_2)\end{aligned}$$



PDL-transformations

$$\begin{aligned} |\Lambda| &= \langle r_1 \rangle p_1 \vee \langle r_2 \rangle p_2 \\ \Lambda(q_1) &= \langle r_2 \rangle \neg p_1 \\ \Lambda(q_2) &= \langle p_2?; r_1 \rangle \neg p_2 \\ \Lambda(s) &= (r_1; r_2) \cup (p_1?; r_2) \end{aligned}$$



PDL Transformations: Signatures

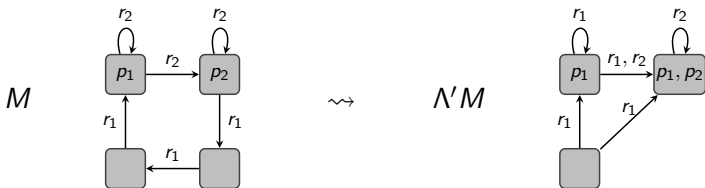
The Λ of our example changed the signature of the model.

M has signature $\{p_1, p_2, r_1, r_2\}$
 ΛM has signature $\{q_1, q_2, s\}$

But we can also define signature-preserving transformations. . .

PDL Transformations: Signature preserving

$$\begin{aligned} |\Lambda'| &= \langle r_1 \rangle p_1 \vee \langle r_2 \rangle p_2 \\ \Lambda'(p_1) &= \langle r_2 \rangle \neg p_1 \\ \Lambda'(p_2) &= \langle p_2?; r_1 \rangle \neg p_2 \\ \Lambda'(r_1) &= (r_1; r_2) \cup (p_1?; r_2) \\ \Lambda'(r_2) &= r_2 \end{aligned}$$



PDL Dynamic Operators

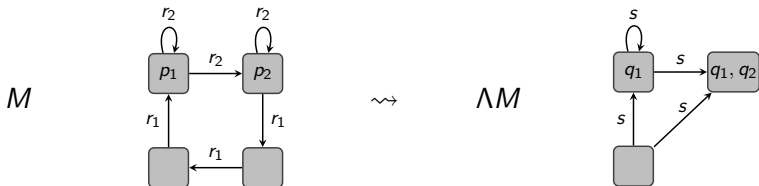
For $u \in |\Lambda|$,

$$M, u \models [\Lambda]\varphi \quad \text{iff} \quad \Lambda M, u \models \varphi$$

PDL Dynamic Operators

For $u \in |\Lambda|$,

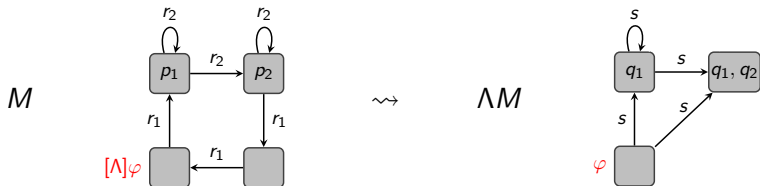
$$M, u \models [\Lambda]\varphi \quad \text{iff} \quad \Lambda M, u \models \varphi$$



PDL Dynamic Operators

For $u \in |\Lambda|$,

$$M, u \models [\Lambda]\varphi \quad \text{iff} \quad \Lambda M, u \models \varphi$$



Note that φ is in the signature of ΛM .

Spies in danger

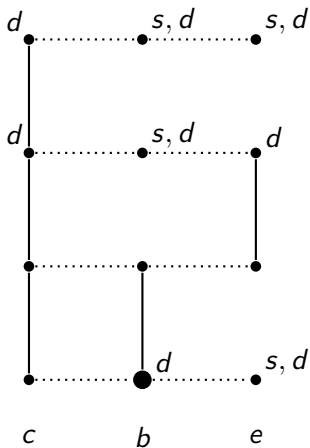
Berlin, 1978. A spy network has recently been uncovered by the Stasi, who are rounding up the spies and their associates. Bella (b) is friends with Charlie (c) and Erik (e), neither of whom are friends with each other. Unknown to the others is that Erik is a spy (s). The others are not spies, and Erik knows that because all spies know who else is a spy (we suppose). Bella knows that Charlie is not a spy, but Charlie does not know about her. After the network is exposed, all the spies and their friends will be interrogated by the police. But just before this happens a message is relayed to all agents revealing whether or not they are in danger, that is, whether they are a spy (which they would know in any case) or a friend of a spy. Who now knows that Erik is a spy?

Spies in danger

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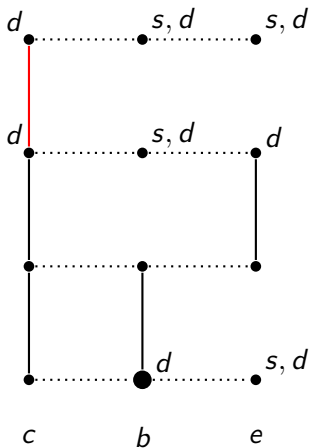
Transforming the spy network model

$$\text{cut}_K(d) = (d?; K; d?) \cup (\neg d?; K; \neg d?)$$



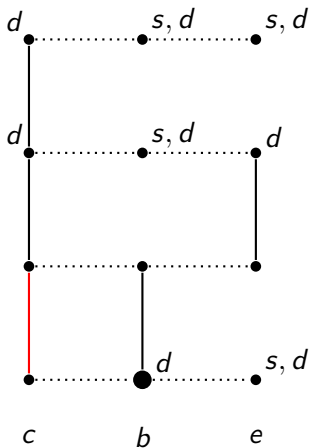
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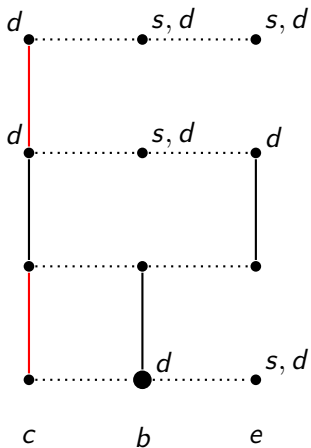
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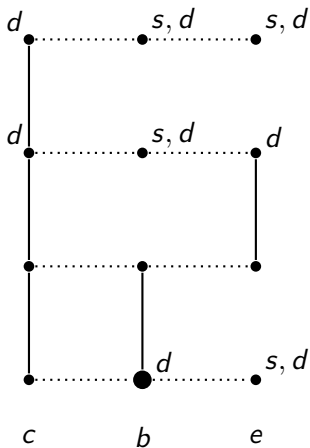
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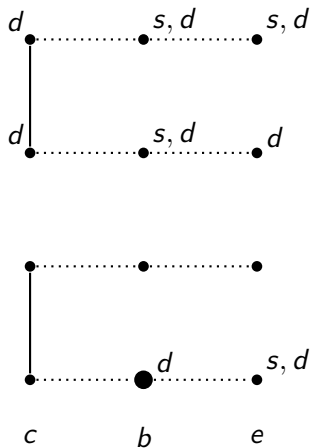
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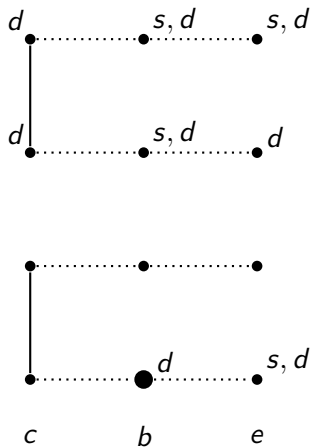
$$[K := \text{cut}_K(d)]$$

\Rightarrow



Transforming the spy network model

Who then knows that Erik is a spy (and didn't know before)?

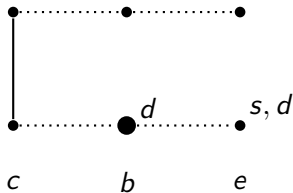
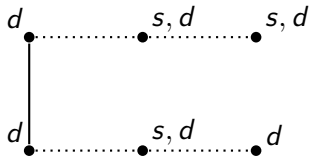


Transforming the spy network model

Who then knows that Erik is a spy (and didn't know before)?

For which n ,

$\mathcal{O}_n(\neg K @_e s \wedge [K := \text{cut}_K(d)] K @_e s)$?



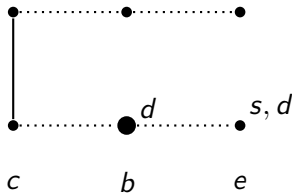
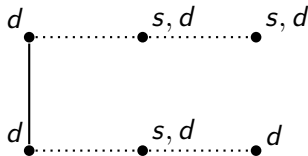
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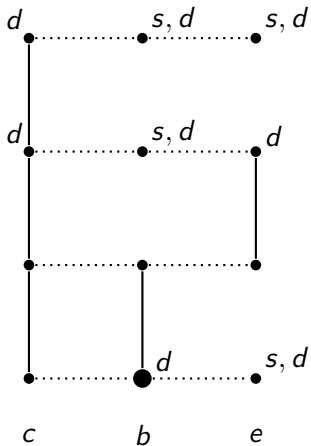
$\mathcal{O}_n(\neg K @_e s \wedge [K := \text{cut}_K(d)] K @_e s)$?

Answer: b (Bella)



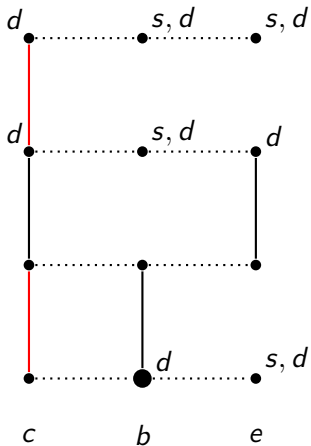
Restricting to Bella's friends

$$\text{send}_{\langle F \rangle b}(d) := [K := (\langle F \rangle b?; \text{cut}_K(d)) \cup (\neg \langle F \rangle b?; K)]$$



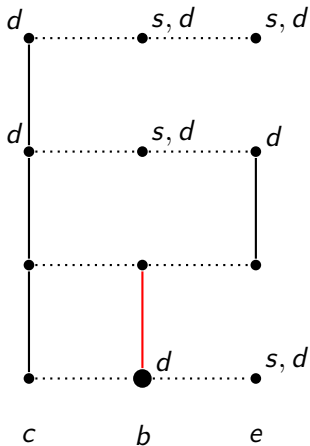
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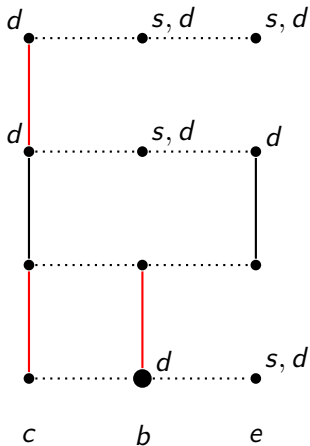
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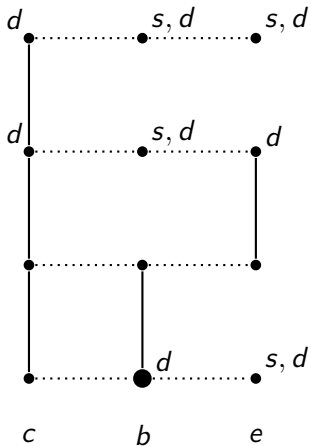
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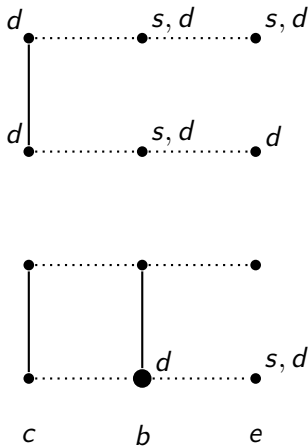
Restricting to Bella's friends

$$\text{send}_{\langle F \rangle b}(d) := [K := (\langle F \rangle b?; \text{cut}_K(d)) \cup (\neg \langle F \rangle b?; K)]$$



$\text{send}_{\langle F \rangle b}(d)$

\Rightarrow



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Sending messages

Revealing whether ψ to agents θ :

$$\text{send}_{\theta}(\psi) = [K := (\theta?; \text{cut}_{\mathcal{K}}(\psi)) \cup (-\theta?; K)]$$

Were η to send the message ψ to agents θ then φ .

$$(\eta \text{ has the information } \psi \rightarrow [\text{send}_{\theta}(\psi)]\varphi)$$

But the details depend on the kind of message being sent. . .

Announcements about the sender

Were η to announce to θ that ψ (about η), then φ .

$$[\eta \triangleleft \psi! : \theta] \varphi = (\@_{\eta} K \psi \rightarrow [\text{send}_{\theta} (@_{\eta} \psi)] \varphi)$$

Announcements about the sender

Were η to announce to θ that ψ (about η), then φ .

$$[\eta \triangleleft \psi! : \theta] \varphi = (\@_{\eta} K \psi \rightarrow [\text{send}_{\theta} (@_{\eta} \psi)] \varphi)$$

Public announcements

$[e \triangleleft s! : \top] AK @_e s$ Were Erik to announce to everyone that he is a spy, then everyone would know he is.

$[b \triangleleft s! : \top] AK @_b s$ Were Bella to announce to everyone that she is a spy, then everyone would know she is.

Announcements about the sender

Were n to announce to θ that ψ (about n), then φ .

$$[n \triangleleft \psi! : \theta] \varphi = (\@_{\eta} K \psi \rightarrow [\text{send}_{\theta}(\@_n \psi)] \varphi)$$

Personal announcements

$$[e \triangleleft s! : b] \@_b K \@_e s$$

Were Erik to announce to Bella that he is a spy, then she would know he is.

$$(\neg(b \vee K \@_e s) \rightarrow [e \triangleleft s! : b] \neg K \@_e s)$$

An agent who is neither Bella nor (already) knows that Erik is a spy, still would not know this after he announces it to Bella.

Announcements about the sender

Were n to announce to θ that ψ (about η), then φ .

$$[\eta \triangleleft \psi! : \theta] \varphi = (\@_{\eta} K \psi \rightarrow [\text{send}_{\theta}(\@_{\eta} \psi)] \varphi)$$

Announcements to friends

$[b \triangleleft \neg s! : \langle F \rangle b] \@_b F K \@_{b \neg s}$ If Bella were to tell her friends that she is not a spy then they would all know that she isn't a spy.

Announcements about the receiver

Were η to announce to θ that ψ (about each of them), then φ .

$$[\eta: \psi! \triangleright \theta]\varphi = (\textcircled{\text{A}}_{\eta} A(\theta \rightarrow \psi) \rightarrow [\text{send}_{\theta}(\psi)]\varphi)$$

Announcements about the receiver

Were η to announce to θ that ψ (about each of them), then φ .

$$[\eta: \psi! \triangleright \theta]\varphi = (\textcircled{\text{A}}_{\eta} A(\theta \rightarrow \psi) \rightarrow [\text{send}_{\theta}(\psi)]\varphi)$$

Public announcements

$\downarrow n [n: d! \triangleright \top]AKd$ Were I to announce to everyone 'you are in danger', everyone would know they are.

Announcements about the receiver

Were η to announce to θ that ψ (about each of them), then φ .

$$[\eta : \psi! \triangleright \theta] \varphi = (\@_{\eta} A(\theta \rightarrow \psi) \rightarrow [\text{send}_{\theta}(\psi)] \varphi)$$

Personal announcements

$[e : d! \triangleright b] \varphi$ Were Erik to announce to Bella, 'you are in danger', then φ .

$[e \triangleleft \@_b d! : b] \varphi$ Were Erik to announce to Bella, 'Bella is in danger', then φ .

Announcements about the receiver

Were η to announce to θ that ψ (about each of them), then φ .

$$[\eta: \psi! \triangleright \theta] \varphi = (\textcircled{\eta} A(\theta \rightarrow \psi) \rightarrow [\text{send}_{\theta}(\psi)] \varphi)$$

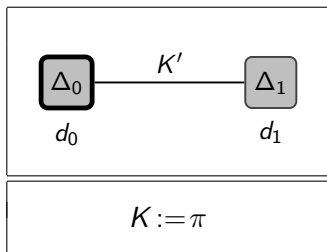
Announcements to friends

$\downarrow n [n: \langle F \rangle n! \triangleright \langle F \rangle n] FK \langle F \rangle n$ Were I to announce to my friends, 'you are my friend', then all my friends would know they are my friends.

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A GDDL Operator



Action d_0 (designated) results in PDL-transformation Δ_0 .

Action d_1 results in PDL-transformation Δ_1 .

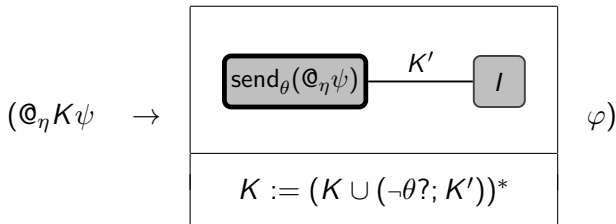
The two results are pointwise linked by K' .

The final result is computed by $K := \pi$

(π is a PDL-program also, possibly, using K').

Private social announcements

$[\eta \triangleleft @_{\eta}\psi! : \theta]$



Were η to send a private message that ψ (about herself) to agents θ , then φ .

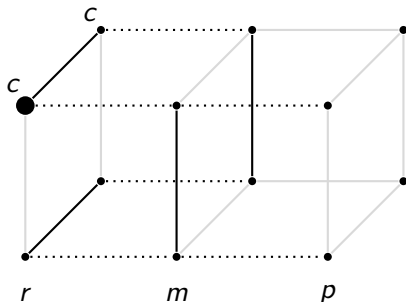
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Roger's Quandry

Peggy (p) knows that Roger (r) is cheating (c) on his wife, Mona (m). What's more, Roger knows that Peggy knows, because they met accidentally while he was with his mistress. Mona does not know about the affair, and both Peggy and Roger know this. The situation (for Roger) deteriorates when he discovers that Peggy is a terrible gossip. She is bound to have told all her friends about his affair. What Roger does not know is whether Mona is a friend of Peggy (she is). Who knows what, exactly?

Roger's Quandary



c

$$\downarrow n K(\textcircled{p}K\textcircled{nc} \wedge \textcircled{m}\neg K\textcircled{nc})$$

$$\downarrow n \textcircled{p}K\textcircled{nc}K\textcircled{p}K\textcircled{nc}$$

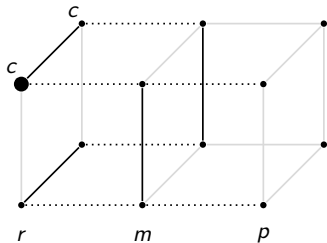
$$(\neg K\textcircled{m}\langle F \rangle p \wedge \neg K\textcircled{m}\neg\langle F \rangle p)$$

$$\downarrow n \textcircled{p}K\textcircled{nc}\neg K\textcircled{m}\langle F \rangle p$$

Roger's Quandary

$$\downarrow n [p \triangleleft @_n c! : \langle F \rangle p] @_m K @_n c$$

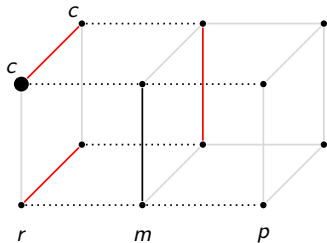
I wouldn't know that Mona would know about my cheating, were Peggy to tell her friends about it.



Roger's Quandary

$$\downarrow n [p \triangleleft @_n c! : \langle F \rangle p] @_m K @_n c$$

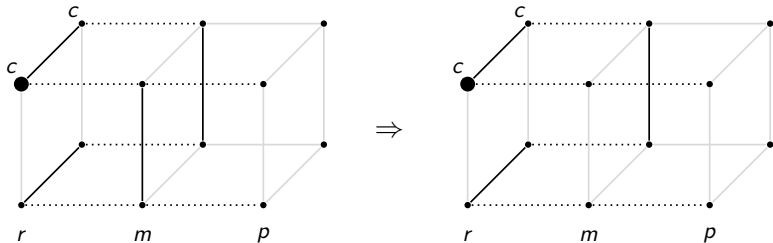
I wouldn't know that Mona would know about my cheating, were Peggy to tell her friends about it.



Roger's Quandary

$$\downarrow n [p \triangleleft @_{nc}! : \langle F \rangle p] @_m K @_n c$$

I wouldn't know that Mona would know about my cheating, were Peggy to tell her friends about it.



Roger needs a little more privacy

Before returning home to face Mona, Roger is uneasy. He would really like to know whether or not she knows about his affair. He already knows that she knows if and only if she is friends with Peggy. So if Peggy told him that they are friends, he would be prepared for Mona's fury. But for his planned excuses to be convincing, Mona must not know that he knows she knows (about the affair). It is therefore very important that Peggy tells him in private.

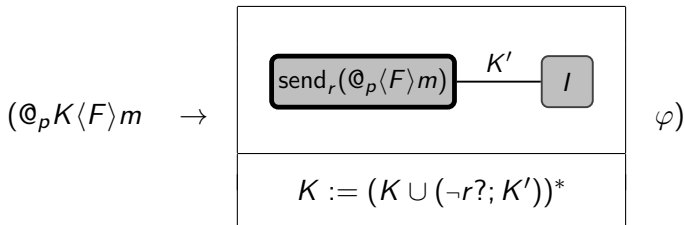
Roger needs a little more privacy

$$\mathbf{[}p \triangleleft \langle F \rangle m! : r\mathbf{]}(\@_r K \@_m K \@_r c \wedge \neg \@_m K \@_r K \@_m K \@_r c)$$

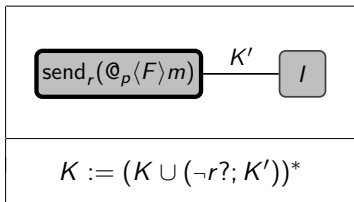
Were Peggy to tell Roger privately that she is friends with Mona, Roger would know Mona knows he has been cheating but Mona wouldn't know that he knows.

Were Peggy to tell Roger, privately

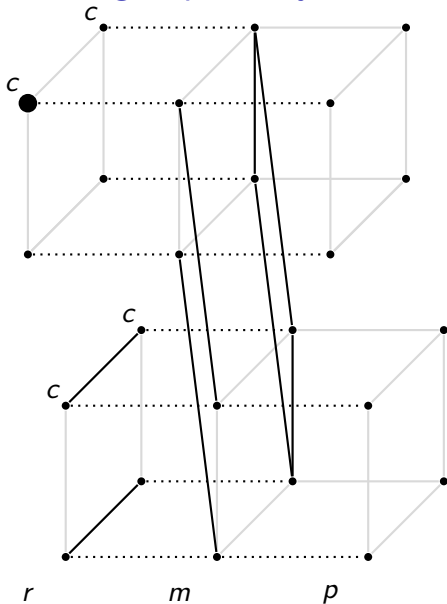
$[p \triangleleft \langle F \rangle m! : r] \varphi$



After Peggy tells Roger, privately



$$(\@_r K \@_m K \@_r c \wedge \neg \@_m K \@_r K \@_m K \@_r c)$$



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Common knowledge reconsidered

It is common knowledge that φ

Common knowledge reconsidered

It is common knowledge that φ

Group identification

Charlie, Bella, and Erik

Bella's friends

Friends of mine

Common knowledge reconsidered

It is common knowledge that φ

Group identification

Charlie, Bella, and Erik

Bella's friends

Friends of mine

Perspective

It is common knowledge that Charlie is not a spy

It is common knowledge among Charlie's friends that I am in danger

It is common knowledge among my friends that they are in danger

Common knowledge, reconsidered

$$\overline{K}_n = (A; n?; K)$$

$[\overline{K}_n]\varphi$ (a PDL formula) is equivalent to $@_n K\varphi$ (an EFL formula)

Common knowledge, reconsidered

$$\overline{K}_n = (A; n?; K)$$

$[\overline{K}_n]\varphi$ (a PDL formula) is equivalent to $@_n K\varphi$ (an EFL formula)

Now, for common knowledge,

$$c_\theta = (A; \theta?; K)^*; A; \theta?$$

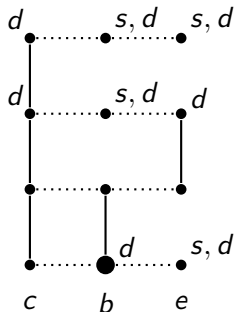
$[c_\theta]\varphi$ means that there is common knowledge among θ -agents that φ .

Common knowledge among enumerated groups

There is common knowledge among Bella and Erik that Bella is not a spy.

$$[c_{(b \vee e)}]@_{b \neg s}$$

$$[(A; (b \vee e)?; K)^*; A; (b \vee e)?]@_{b \neg s}$$

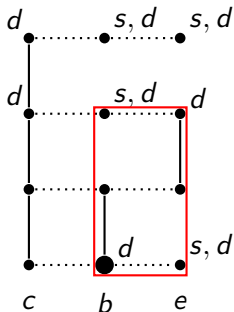


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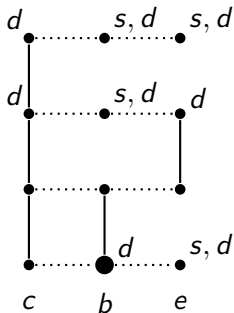


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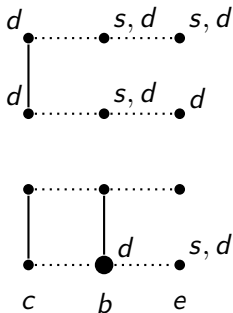
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send _{$\langle F \rangle_b$} (d)

⇒

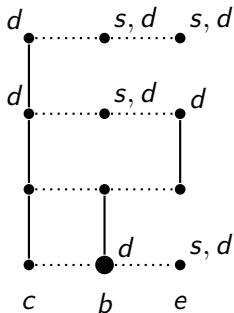


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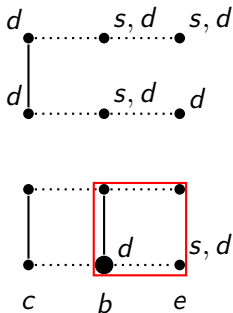
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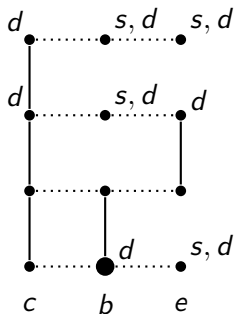


Common knowledge among non-indexically described groups

There is common knowledge among Bella's friends that Bella is not a spy.

$$[c_{\langle F \rangle b}]@_{b^{-s}}$$

$$[(A; \langle F \rangle b?; K)^*; A; \langle F \rangle b?]@_{b^{-s}}$$

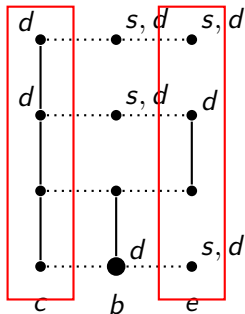


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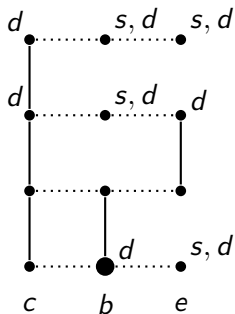


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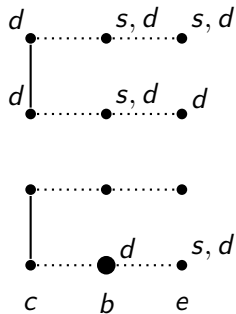
$$[c_{\langle F \rangle b}]@_{b^{-s}}$$

$$[(A; \langle F \rangle b?; K)^*; A; \langle F \rangle b?]@_{b^{-s}}$$



$\text{send}_{\top}(d)$

\Rightarrow

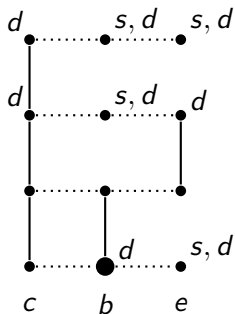


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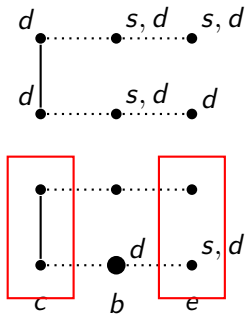
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send_T(d)

⇒

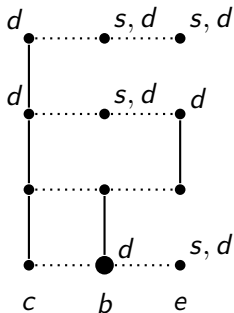


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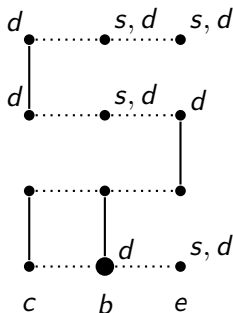
$$[c \langle F \rangle b] @_{b \neg s}$$

$$[(A; \langle F \rangle b?; K)^*; A; \langle F \rangle b?] @_{b \neg s}$$



send_c(d)

\Rightarrow

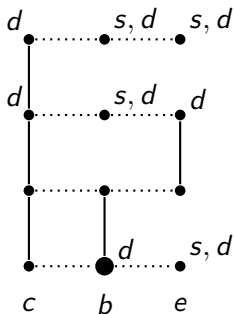


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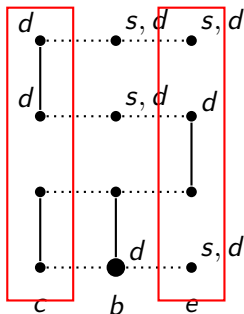
$$[c_{\langle F \rangle b}] @_{b \neg s}$$

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$\text{send}_c(d)$

\Rightarrow

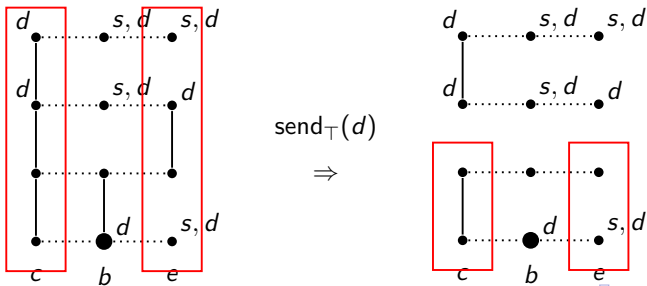


Common knowledge among an indexically described group of agents about an indexical proposition.

There is common knowledge among my friends that they are not friends of a spy.

$$\downarrow n [c_{\langle F \rangle n}] \neg \langle F \rangle s$$

$$\downarrow n [(A; \langle F \rangle n?; K)^*; A; \langle F \rangle n?] \neg \langle F \rangle s$$



Common knowledge, reconsidered

The interaction between indexical descriptions of groups, and indexical contents of propositional attitudes give rise to numerous logical distinctions regarding common knowledge, and its relationship to communication.

Outline

- 1 Introduction
- 2 Epistemic logic: a quick summary
- 3 Epistemic logic of friendship
- 4 PDL-transformations: a quick summary
- 5 Social announcements
- 6 GDDL and privacy
- 7 Knowing your friends
- 8 Common knowledge reconsidered
- 9 Conclusion**

So what?

What is EFL good for?

- an appreciation of the diversity of subtle logic distinctions when combining epistemic and social relations
- a model of reasoning about communication and actions that change the social structure
- relational analyses of belief and preference fit well into this framework

Some References I